Supersymmetric hybrid inflation with minimal and non-minimal Kähler potential

2007 J. Phys. A: Math. Theor. 40 6859

(http://iopscience.iop.org/1751-8121/40/25/S30)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **40** (2007) 6859–6864 doi:10.1088/1751-8113/40/25/S30

Supersymmetric hybrid inflation with minimal and non-minimal Kähler potential

M Bastero-Gil

Departamento de Fisica Teorica y del Cosmos and Centro Andaluz de Fisica de Particulas Elementales, Universidad de Granada, E-19071 Granada, Spain

E-mail: mbg@ugr.es

Received 30 October 2006, in final form 26 February 2007
Published 6 June 2007
Online at stacks.iop.org/JPhysA/40/6859

Abstract

Minimal supersymmetric hybrid inflation based on a minimal Kähler potential predicts a spectral index $n_s \gtrsim 0.98$. On the other hand, WMAP three-year data prefer a central value $n_s \approx 0.95$. We propose a class of supersymmetric hybrid inflation models based on the same minimal superpotential but with a non-minimal Kähler potential. Including radiative corrections using the one-loop effective potential, we show that the prediction for the spectral index is sensitive to the small non-minimal corrections, and can lead to a significantly red-tilted spectrum, in agreement with WMAP.

PACS numbers: 98.80.Cp, 11.30.Pb, 12.60.Jv, 04.65.+e

1. Introduction

Hybrid inflation models [1–3] are examples of small field inflation models which typically predict an approximately scale invariant spectral index and a very small tensor fraction. For such models, the WMAP three-year central value for the spectral index is about $n_s \approx 0.95$ [4], whereas the joint analysis of Ly-α forest power spectrum from the Sloan Digital Sky Survey, with cosmic microwave background, galaxy clustering and supernovae yields $n_s = 0.965 \pm 0.012$ [5]. Consequently, hybrid inflation models which predict the spectral index to be too large are now less preferred.

Amongst these models are those based on minimal supersymmetric hybrid inflation, defined by the superpotential W,

$$\mathcal{W} = \kappa \hat{S}(\hat{\phi}\hat{\bar{\phi}} - M^2), \tag{1}$$

where $\hat{S}$ is a gauge singlet and $\hat{\phi}$, $\hat{\bar{\phi}}$ are a conjugate pair of superfields transforming as non-trivial representations of some gauge group G, together with a minimal Kähler potential

$$\mathcal{K}_0 = |S|^2 + |\phi|^2 + |\bar{\phi}|^2, \tag{2}$$

with $S, \phi, \bar{\phi}$ being the bosonic components of the superfields. The gauge singlet S is a natural candidate for the inflaton in this model. During inflation, the theory is in a false vacuum where $\langle\phi\rangle = \langle\bar{\phi}\rangle = 0$ and $\langle S\rangle \neq 0$, driving inflation. Inflation ends when the field value of the inflaton S falls below some critical value which corresponds to a tachyonic instability for $\langle\phi\rangle$ and/or $\langle\bar{\phi}\rangle$. Inflation ends by a phase transition to the true supersymmetric minimum, with ϕ and $\bar{\phi}$ getting equal non-zero vacuum expectation values (vevs) $\langle\phi\rangle = \langle\bar{\phi}\rangle = M$ whereas $\langle S\rangle = 0$ (or $\mathcal{O}(m_{3/2})$ in broken supersymmetry). In this minimal model, the vacuum expectation values (vevs) $\langle\phi\rangle$ and $\langle\bar{\phi}\rangle$ break G to some subgroup H. If $\phi, \bar{\phi}$ break, e.g. Pati–Salam or $SO(10)$, topological defects such as cosmic strings and/or monopoles are generated after inflation. In order to avoid the monopole problem, one can extend superpotential to so-called shifted or smooth inflation [6]; but here we shall restrict ourselves to the minimal W above.

The theory defined above in equations (1) and (2) defines the minimal supersymmetric F-term hybrid inflation model, which we briefly revise in section 2. On the other hand, there is no symmetry that protects the minimal form of the Kähler potential. In this paper, we study supersymmetric F-term hybrid inflation with non-minimal Kähler potential, including radiative corrections using the one-loop effective potential, and show that the prediction of the spectral index is sensitive to such non-minimal effects, which can lead to a significantly red-tilted spectrum. This is done in section 3.

2. Minimal Kähler potential

In supersymmetric theories based on supergravity (sugra), there is a well-known problem that $\eta \approx 1$ due to the sugra corrections, thereby violating one of the slow roll conditions. The slow-roll parameters may be defined as

$$\epsilon = \frac{m_{\rm P}^2}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta = m_{\rm P}^2\left(\frac{V''}{V}\right), \tag{3}$$

where $m_{\rm P} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass and V', V'' are respectively the first and second derivatives of potential with respect to the inflaton field. Sugra corrections typically induce scalar squared masses of the order of the Hubble constant squared $H^2 = V/3m_{\rm P}^2$, and leading therefore to the so-called η problem [2, 7]. It is an interesting fact that the supergravity potential based on the minimal supersymmetric hybrid inflation theory defined in equations (1), (2) provides a solution to the η problem since the mass squared of the inflaton when calculated from the supergravity potential cancels at the tree level [2]. Nevertheless, sugra corrections will induce quartic and higher order terms in the potential [8].

Therefore, in minimal supersymmetric hybrid inflation the curvature of the potential is given by the one-loop effective potential, $\Delta\mathcal{V}_{1\,\rm loop}$ [3]. During inflation when $|S| > |S^c| = M$, the waterfall field ϕ is held at zero due to having a large positive mass squared, and effectively during inflation we are left with the potential

$$V = V_1^{\rm min}(\phi = 0) \simeq \kappa^2 M^4 \left(1 + \frac{S_R^4}{8m_{\rm P}^4} + \cdots\right) + \Delta\mathcal{V}_{1\,\rm loop}, \tag{4}$$

where $S_R = \sqrt{2}|S|$. When $S_R/M > 1$, the one-loop effective potential can be approximated by $\Delta\mathcal{V}_{1\,\rm loop} \simeq [(\kappa M)^4\mathcal{N}/(4\pi^2)] \ln S_R/Q$, $\mathcal{N}$ being the dimensionality of the representation of the fields $\phi, \bar{\phi}$ and Q is the renormalization scale[1]. The slow-roll parameters are then given by

$$\eta \simeq -\delta \simeq -\frac{\kappa^2\mathcal{N}}{8\pi^2}\left(\frac{m_{\rm P}}{S_R}\right)^2, \qquad \epsilon \simeq \frac{\kappa^2\mathcal{N}}{(4\pi)^2}\delta \ll |\eta|, \tag{5}$$

[1] None of the derivatives of $\Delta\mathcal{V}_{1\,\rm loop}$ depend on the renormalization scale Q, and therefore it would have no effect on the inflationary predictions.

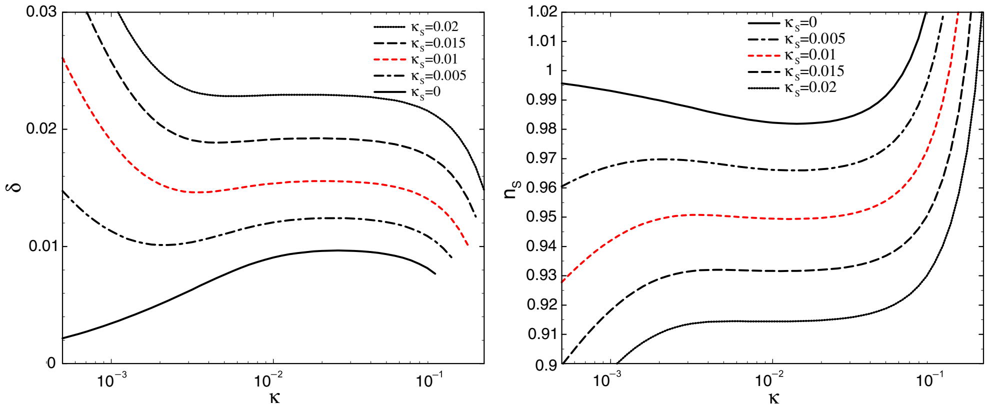

Figure 1. Non-Minimal Kähler potential: (a) (left hand side) predicted value of the one-loop contribution to the spectral index, δ, and (b) (right hand side) predicted value of the spectral index n_s depending on the value of the coupling κ, for different values of $\kappa_S = 0.02, 0.015, 0.01, 0.005, 0$. $(\mathcal{N} = 1)$.

where we have denoted by δ the contribution to η from the effective potential. Hybrid inflation ends when the value of the inflaton field reaches the critical value, and integrating back the evolution equations we can obtain the value of the field N_e e-folds before the end of inflation, which again when $S_{Re}/M > 1$ can be approximated by $S_{Re} \simeq \sqrt{N_e\mathcal{N}}\kappa m_{\rm P}/(2\pi)$. Given the value of the field, the amplitude of the primordial spectrum is given by

$$P_{\mathcal{R}}^{1/2} \simeq \frac{V}{V'}\left(\frac{H}{2\pi m_{\rm P}^2}\right) \simeq 2\sqrt{\frac{N_e}{3\mathcal{N}}}\left(\frac{M}{m_{\rm P}}\right)^2. \tag{6}$$

The WMAP normalization is $P_{\mathcal{R}}^{1/2} = 4.86 \times 10^{-5}$, taken at the comoving scale $k_0 = 0.002\ \mathrm{Mpc}^{-1}$, which corresponds to $N_e \approx 50$ [9, 10]. From equation (6) this fixes the inflationary scale $M \simeq 6 \times 10^{15}$ GeV. And for the spectral index, $n_s \simeq 1 + 2\eta - 6\epsilon$, we have the approximated result [3]

$$n_s \simeq 1 - 2\delta \simeq 1 - \frac{1}{N_e} \simeq 0.98, \tag{7}$$

for $N_e \approx 50$. The tensor to scalar ratio is negligible, with $r \lesssim 10^{-4}$, and also there is no running in the spectral index, with $\mathrm{d}n_s/\mathrm{d}\ln k \lesssim 10^{-3}$ [11].

The predicted value of the spectral index deviates from the approximated value equation (7) for small and large values of κ (see figure 1(*b*), solid line). For small values of the coupling κ, the approximation $S_{Re}/M > 1$ does not hold. Diminishing the coupling what we have is a flatter potential, with a smaller curvature, so that the last say 50 e-folds of inflation happens to be quite close to the critical value, giving rise to a practically scale invariant spectrum. On the other hand, for larger values of the coupling κ, the value S_{Re} gets larger and closer to the Planck scale, so that the quartic term for the inflaton induced by the sugra corrections can no longer be neglected. This tends to give a positive curvature contribution, making the spectrum to turn from red tilted ($n_s < 1$) to blue tilted ($n_s > 1$) [8]. Therefore, the result in equation (7) can be viewed as the lower bound on the predicted spectral index, with $n_s \gtrsim 0.98$, to be compared to the central WMAP three-year central value of $n_s \approx 0.95$.

The latter is the preferred value with no tensor and no cosmic string contributions to the primordial spectrum. In this kind of models the tensor contribution is typically suppressed, but we may have a non-negligible contribution of cosmic strings [12], depending on details of the model. In that case, they cannot contribute more than a 10% to the total, but their effect on the primordial spectrum translates into larger allowed values for the spectral index, and in particular in [13] it is shown that allowing a contribution of around 5% of cosmic strings in the analysis can increase the preferred value for the spectral index up to $n_s \approx 0.98$.

On the other hand, with no tensor and no cosmic strings, the predicted spectral index can be lowered by considering a non-minimal Kähler potential, as we shall see in the following.

3. Non-minimal Kähler potential

We now turn to the non-minimal modification of supersymmetric hybrid inflation [14]. We continue to assume the same minimal superpotential as in equation (1). However, we now consider a non-minimal Kähler potential[2], [16, 17],

$$\mathcal{K} = |S|^2 + |\phi|^2 + |\bar{\phi}|^2 + \kappa_S \frac{|S|^4}{4m_P^2} + \kappa_{S\phi}\frac{|S|^2|\phi|^2}{m_P^2} + \kappa_{S\bar{\phi}}\frac{|S|^2|\bar{\phi}|^2}{m_P^2} + \kappa_{SS}\frac{|S|^6}{6m_P^4} + \cdots . \quad (8)$$

As we will see below, the inflaton gets now a mass squared proportional to $3\kappa_S H^2$, with $\eta \simeq -\kappa_S + \cdots$, so the first constraint we must impose on our expansion parameters is having $\kappa_S < 1$, which is just the well-known η problem. In our phenomenological approach, we consider the coefficients in equation (8) as free parameters, using cosmological observations to constrain their values. In order to have enough inflation and the spectral index in the range allowed by observations we will require (see figure 1(*b*)) $\kappa_S \lesssim 0.02$, which is a more severe constraint than that imposed by the η problem. This sensitivity is an interesting result in itself, but it does raise the theoretical question of the origin of such small coefficients multiplying the non-minimal corrections to the Kähler potential. A possible theoretical motivation for having a canonical Kähler potential with small corrections is provided by the proposal of Watari and Yanagida [18]. In this paper, they consider the same superpotential as in our equation (1) and show by imposing a global $\mathcal{N} = 2$ supersymmetry and the usual $U(1)_R$ symmetry that the resulting Kähler potential has a leading order canonical form, with additional non-minimal corrections arising from radiative corrections with naturally suppressed coefficients, as we assume here. Therefore, although our present analysis is mainly motivated by phenomenological considerations, we believe that the canonical form of the Kähler potential, supplemented by non-minimal corrections with small coefficients, has also some theoretical motivation.

Assuming the Kähler potential in equation (8) and keeping the relevant terms for inflation up to $O((|S|/m_P)^4)$, we find the potential[3]

$$V \simeq \kappa^2 M^4 \left(1 - \kappa_S \frac{S_R^2}{2m_P^2} + \gamma_S \frac{S_R^4}{8m_P^4} + \cdots\right) + \Delta\mathcal{V}_{1\,\mathrm{loop}}, \quad (9)$$

where $\gamma_S = 1 - 7\kappa_S/2 + 2\kappa_S^2 - 3\kappa_{SS}$. Although it seems that we are introducing an infinite number of arbitrary parameters in the expansion of the Kähler potential, equation (8), we remark that in the regime where the inflaton field value is well below the Planck mass, the non-minimal Kähler contributions to the quartic and higher terms for the inflaton have no effect on the inflationary dynamics and therefore only one parameter, κ_S, will be relevant for

[2] Non-minimal corrections to the Kähler potential have also been considered in D-term hybrid inflation in [15].

[3] The non-minimal Kähler only introduces a small correction to the ϕ squared mass, so that still for values of the inflaton field $|S| > |S|^c$ this is positive and we can set $\phi = 0$ during inflation.

the inflationary predictions that follow. Note that $\kappa_S > 0$ will be required so that the prediction for n_s is in agreement with WMAP.

The non-minimal Kähler induces now a negative correction to both the first and the second derivatives of the potential in the inflaton direction:

$$V' \simeq \frac{\kappa^2 M^4}{m_{\mathrm{P}}} \left(-\kappa_S \frac{S_R}{m_{\mathrm{P}}} + \gamma_S \frac{S_R^3}{2m_{\mathrm{P}}^3} + \frac{\kappa^2 \mathcal{N}}{8\pi^2} \frac{m_{\mathrm{P}}}{S_R} \right), \tag{10}$$

$$V'' \simeq \frac{\kappa^2 M^4}{m_{\mathrm{P}}^2} \left(-\kappa_S + 3\gamma_S \frac{S_R^2}{2m_{\mathrm{P}}^2} - \frac{\kappa^2 \mathcal{N}}{8\pi^2} \frac{m_{\mathrm{P}}^2}{S_R^2} \right). \tag{11}$$

This correction gives rise to a local minimum and maximum in the potential located at

$$\frac{S_R^{\mathrm{min}}}{m_{\mathrm{P}}} \simeq \sqrt{\frac{2\kappa_S}{\gamma_S}}, \qquad \frac{S_R^{\mathrm{max}}}{m_{\mathrm{P}}} \simeq \sqrt{\frac{2\mathcal{N}}{\kappa_S}} \left(\frac{\kappa}{4\pi} \right), \tag{12}$$

which for example for $\kappa_S \simeq \kappa \simeq 0.01$ gives $S_R^{\mathrm{min}}/m_{\mathrm{P}} \simeq 0.14$ and $S_R^{\mathrm{max}}/m_{\mathrm{P}} \simeq 0.01$. After that, for $S_R < S_R^{\mathrm{max}}$ we have the standard flat potential with $V' > 0$, suitable for hybrid inflation, with the field rolling towards the critical value. We have demanded then that we can get at least $60 - 50$ e-folds of inflation, once the field is in that region of the potential with $V' > 0$, i.e., that $S_{Re} \leqslant S_R^{\mathrm{max}}$. We do not address the question of how the field reaches S_{Re} in this paper, that is, the problem of the initial conditions for inflation. Here we just concentrate on the inflationary predictions derived from the potential equation (9), assuming that we have suitable initial conditions for hybrid inflation to take place. The condition of having enough inflation, $S_{Re} \leqslant S_R^{\mathrm{max}}$, would give us an upper bound on the possible value of κ_S. However, the value of the field at N_e e-folds, S_{Re}, itself depends on the value of κ_S through V'. The contribution from κ_S tends to decrease V' and makes the potential flatter, so that the corresponding value of S_{Re} decreases and it will stay below S_R^{max}.

In addition, we have now in equation (9) a mass term for the inflaton field proportional to κ_S, and therefore this parameter has to be small enough in order to satisfy the slow-roll conditions. Taking $S_R \ll m_{\mathrm{P}}$, so that we can neglect the quartic term in the analytical expression[4], we have

$$\eta \simeq -\kappa_S - \delta, \tag{13}$$

where δ is the contribution from the one-loop effective potential, and again $\epsilon \ll |\eta|$. Therefore, for slow-roll inflation, $|\eta| < 1$, we only require $\kappa_S < 1$. The spectral index is then given by

$$n_s \simeq 1 - 2\kappa_S - 2\delta. \tag{14}$$

From the previous analysis with minimal Kähler potential, we could think naively that the one-loop contribution $\delta \leqslant 0.01$, and then we would need for example $\kappa_S \geqslant 0.01$ if we want the spectral index around or below $n_s \approx 0.96$. However, as previously noted, the non-minimal Kähler contribution will decrease V' which, from equation (6), tends to increase the amplitude of the curvature perturbation. Thus, in order to keep the WMAP normalization, the scale of inflation M (i.e. V) has to decrease accordingly. Also, a decrease in V' means a smaller value of the field at 50 e-folds, which implies a larger value of $\delta \sim -(\kappa^2\mathcal{N}/(8\pi^2))(m_{\mathrm{P}}/S_R)^2$. Therefore, when taking into account the effects of the non-minimal Kähler potential we have also that the one-loop contribution can be well above the previous upper limit of 0.01. Note that there is no regime where the one-loop contribution could be neglected with respect to the non-minimal Kähler one, as far as the approximation $S_R < m_{\mathrm{P}}$ is fulfilled. This can clearly

[4] The quartic term for the inflaton is taken into account in all the numerical calculations, and therefore in the results presented in the plots.

be seen in figure 1(*a*) where we show the one-loop contribution to the spectral index, δ, for different values of κ_S. The general trend is that the one-loop effective potential contribution always remains non-negligible, and besides $\delta > \kappa_S$.

In figure 1(*b*), we plot the prediction for n_s as a function of κ for different values of κ_S. We can see that even for small values of κ, already for $\kappa_S \simeq 5 \times 10^{-3}$, we obtain a spectral index smaller than what we would have expected only from the non-minimal Kähler contribution, due to the increase in δ. As the value of κ_S increases, the effect gets larger and the spectrum more and more red-tilted. However, for a given value of κ_S, the prediction for the spectral index is practically independent of the value of κ, for values of the coupling in the range [0.001, 0.05].

On the other hand, as we increase the coupling κ, the field value at 50 e-folds also increases and approaches the Planck scale. The quartic term in the potential then takes over and gives rise to a blue-tilted spectrum, just as with the minimal Kähler potential. At which value of κ this effect dominates depends on the value of the quartic coefficient γ_S, which in turn may depend now also on the next parameter in the expansion of the non-minimal Kähler potential, i.e. κ_{SS}. Nevertheless, for values of $\kappa_{SS} < 1/3$, this parameter has no effect on the spectral index.

In summary, we have argued that a relatively modest extension of minimal supersymmetric hybrid inflation preserves many of its successful features and also yields a scalar spectral index which appears to be more consistent with the most recent data, when no tensor and no cosmic strings contributions are included. For values of the couplings $\kappa \approx \kappa_S \approx 0.01$ we obtain $n_s \simeq 0.95$, which are consistent with the analysis done also in [13] when including non-minimal sugra corrections.

References

[1] Linde A D 1990 *Phys. Lett.* B **249** 18
Linde A D 1991 *Phys. Lett.* B **251** 38

[2] Copeland E J, Liddle A R, Lyth D H, Stewart E D and Wands D 1994 *Phys. Rev.* D **49** 6410

[3] Dvali G R, Shafi Q and Schaefer R K 1994 *Phys. Rev. Lett.* **73** 1886

[4] Spergel D N *et al* 2006 *Preprint* astro-ph/0603449
Kinney W H, Kolb E W, Melchiorri A and Riotto A 2006 *Preprint* astro-ph/0605338

[5] Seljak U, Slosar A and McDonald P 2006 *Preprint* astro-ph/0604335

[6] For a review on standard, smooth and shifted hybrid inflation see: Şenoğuz V N and Shafi Q 2005 *Preprint* hep-ph/0512170 and references therein

[7] Dine M, Randall L and Thomas S 1995 *Phys. Rev. Lett.* **75** 398

[8] Panagiotakopoulos C 1997 *Phys. Rev.* D **55** 7335
Linde A D and Riotto A 1997 *Phys. Rev.* D **56** 1841

[9] Leach S M and Liddle A R 2003 *Phys. Rev.* D **68** 123508

[10] Şenoğuz V N and Shafi Q 2005 *Phys. Rev.* D **71** 043514

[11] Şenoğuz V N and Shafi Q 2003 *Phys. Lett.* B **567** 79

[12] Jeannerot R, Rocher J and Sakellariodou M 2003 *Phys. Rev.* D **68** 103514

[13] Battye R A, Garbrecht B and Moss A 2006 *Preprint* astro-ph/0607339

[14] Bastero-Gil M, King S F and Shafi Q 2006 *Preprint* hep-ph/0604198

[15] Seto O and Yokoyama J 2006 *Phys. Rev.* D **73** 023508
Lin C-M and McDonald J 2006 *Preprint* hep-ph/0604245

[16] Şenoğuz V N and Shafi Q 2004 *Phys. Lett.* B **596** 8

[17] Antusch S, Bastero-Gil M, King S F and Shafi Q 2005 *Phys. Rev.* D **71** 083519

[18] Watari T and Yanagida T 2001 *Phys. Lett.* B **499** 297